

A Note on "M – component nonlinear evolution equations: multiple soliton solutions"

Naum N. Muraved

Institute of Computer Science, National Research Nuclear University MEPhI, 31 Kashirskoe Shosse, 115409 Moscow, Russian Federation

Abstract

We analyze the recent paper by Wazwaz [Wazwaz A.M., M – component nonlinear evolution equations: multiple soliton solutions, Phys. Scr. 81 (2010) 055004]. We demonstrate that author did not consider in essence the M – component nonlinear evolution equations but he reduced the M – component equations to the well – known Korteweg – de Vries equation, to modified Korteweg – de Vries equation and to the Kadomtsev – Petviashvili equation. To find multiple soliton solutions for these well – known equations author has used the Hirota method.

Recently Wazwaz [1] studied the four M – component nonlinear evolution equations, namely the M – component Korteweg – de Vries (KdV) equation

$$\frac{\partial u_i}{\partial t} + \alpha_i \left(\sum_{k=1}^M u_k \right) \frac{\partial u_i}{\partial x} + \frac{\partial^3 u_i}{\partial x^3} = 0, \quad (1)$$

the M – component Kadomtsev – Petviashvili equation

$$\left(\frac{\partial u_i}{\partial t} + \alpha_i \left(\sum_{k=1}^M u_k \right) \frac{\partial u_i}{\partial x} + \frac{\partial^3 u_i}{\partial x^3} \right)_x + \frac{\partial^2 u_i}{\partial y^2} = 0, \quad (2)$$

the M – component mKdV equation

$$\frac{\partial u_i}{\partial t} + \alpha_i \left(\sum_{k=1}^M u_k^2 \right) \frac{\partial u_i}{\partial x} + \frac{\partial^3 u_i}{\partial x^3} = 0, \quad (3)$$

and the M – component mKdV – KP equation

$$\left(\frac{\partial u_i}{\partial t} + \alpha_i \left(\sum_{k=1}^M u_k^2 \right) \frac{\partial u_i}{\partial x} + \frac{\partial^3 u_i}{\partial x^3} \right)_x + \frac{\partial^2 u_i}{\partial y^2} = 0. \quad (4)$$

Firstly author [1] has aimed "to show that a variety of M – component nonlinear evolution equations belong to the class of integrable equations". Secondly author [1] has sought "to determine multiple soliton solutions and multiple – singular soliton solutions for these equations".

The aim of this note is to show that author [1] has not considered the M – component nonlinear evolution equations. We demonstrate that using the additional condition for components u_k author [1] reduced the M – component nonlinear evolution equations to the well – known integrable equations.

Let us demonstrate this fact using the system of equations (1). Author [1] looked for solutions of this system assuming

$$u_i = R_i (\ln f)_{xx}, \quad (5)$$

From condition (5) we obtain the following equalities

$$\frac{u_1}{R_1} = \frac{u_2}{R_2} = \dots = \frac{u_k}{R_k} = \dots = \frac{u_M}{R_M} = (\ln f)_{xx}, \quad (6)$$

Taking equations (6) into account we can present equation (1) in the form

$$\frac{\partial u_i}{\partial t} + \frac{\alpha_i}{R_i} \left(\sum_{k=1}^M R_k \right) u_i \frac{\partial u_i}{\partial x} + \frac{\partial^3 u_i}{\partial x^3} = 0, \quad (7)$$

Assuming in (6)

$$u_i = \frac{6R_i}{\alpha_i \sum_{k=1}^M R_k} u \quad (8)$$

we have the well – known Korteweg – de Vries equation in the form

$$u_t + 6 u u_x + u_{xxx} = 0 \quad (9)$$

Note that equation (8) is the famous Korteweg - de Vries equation [2–4]. There are soliton solutions of this equation [5,6] that can be obtained by the Hirota method [7] taking the following formula into consideration

$$u = 2 \frac{\partial^2 \ln f}{\partial x^2} \quad (10)$$

As consequence of this observation we obtain that author [1] did not study the system of equations (1) but he looked for solution of the following system of equations

$$u_t + 6 u u_x + u_{xxx} = 0, \quad u_i = \frac{6R_i}{\alpha_i \sum_{k=1}^M R_k} u \quad (11)$$

The second equation is trivial algebraic equations for finding u_i . No doubt the system of equations (11) is the completely integrable system but we do not see the subject of publication in this direction. We can suggest a lot of similar "completely integrable system of equations".

Let us note that using the formula (8) we can reduce (2) to the well - known Kadomtsev – Petviashvili equation

$$\left(\frac{\partial u}{\partial t} + 6 u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} \right)_x + \frac{\partial^2 u}{\partial y^2} = 0. \quad (12)$$

Soliton solutions of equation (12) are well known [8].

Assuming in (3)

$$u_i = \frac{R_i^2}{\alpha_i \sum_{k=1}^M R_k^2} v \quad (13)$$

we obtain the modified Korteweg – de Vries equation

$$\frac{\partial v}{\partial t} + v^2 \frac{\partial v}{\partial x} + \frac{\partial^3 v}{\partial x^3} = 0. \quad (14)$$

Soliton solutions of (14) are well known as well [9].

The the system of equation (4) at $y = x$ can be reduced to the mKdV equation as well.

Unfortunately the author [1] does not present new results to the problem of integrability of systems (1), (2), (3) and (4) except trivial exercise on the application of the Hirota method to the famous nonlinear integrable equations. He has made the continuation of the errors that were discussed in recent papers [10–20].

References

- [1] Wazwaz A.M. M – component nonlinear evolution equations: multiple soliton solutions, Phys Scr, 81 (2010) 055004 (7 pp)

- [2] Korteweg D.J., de Vries G. On the change of form of long waves advancing in a rectangular canal and on a new tipe of long stationary waves. Phil. Mag. 39 (1895) 422 - 43
- [3] Zabuski N.J., Kruskal M.D., Integration of "solitons" in a collisionless plasma and the recurrence of initial states, Phys. Rev. Lett. 15 (1965) 240 - 42
- [4] Ablowitz M.J. and Clarkson P.A., Solitons Nonlinear Evolution Equations and Inverse Scattering, Cambridge university press, 1991
- [5] Gardner C.S., Greene J.M., Kruskal M.D., Miura R.M., Method for solving the Korteweg - de Vries equation, J. Math. Phys., 19 (1967), 1095 - 1097
- [6] Lax P.D., Integrals of nonlinear equations of evolution and solitary waves, Comm. Pure Appl. Math., v.21, (1968), 467 - 490
- [7] Hirota R., Exact solution of the Korteweg-de Vries for multiple collisions of solutions, Phys. Rev. Lett., 27 (1971) 1192 - 94
- [8] Satsuma J., N - soliton solution of the two - dimensional Kprteweg - de Vries equation, 40 (1976), 286 - 290
- [9] Ablowitz M.J., Segur H., Solitons and the Inverse Scattering Transform, SIAM, Philadelphia, 1981
- [10] Kudryashov N.A., Loguinova N.B., Extended simplest equation method for nonlinear differential equations, Applied Mathematics and Computation, 205 (2008) 396 - 402
- [11] Kudryashov N.A., Loguinova N.B., Be careful with Exp-function method, Commun. Nonlinear Sci. Numer. Simulat. 2009;14:1881 - 1890
- [12] Kudryashov N.A., On "new travelling wave solutions" of the KdV and the KdV-Burgers equations. Commun Nonlinear Sci Numer Simul 2009;14:1891-900.
- [13] Kudryashov N.A., Seven common errors in finding exact solutions of nonlinear differential equations, Commun Nonlinear Sci Numer Simul 2009;14:35033529.
- [14] Kudryashov N.A., Soukharev M.B. , Popular Ansatz methods and Solitary wave solutions of the Kuramoto-Sivashinsky equation, Regular and Chaotic Dynamics, 14 (2009) 407 - 419

- [15] Kudryashov N.A., Comment on: "A novel approach for solving the Fisher equation using Exp-function method", Physics Letters A 2009;373:1196 - 1197
- [16] Kudryashov N.A., Soukharev M.B., Multi soliton solution, rational solution of the Boussinesq – burgers equations, Commun Nonlinear Sci Numer Simulat 15 (2010), 1765 – 1767
- [17] Kudryashov N.A. Meromorphic solutions of nonlinear differential equations, Commun Nonlinear Sci Numer Simulat 15 (2010), 2778 – 2790
- [18] Kudryashov N.A., Ryabov P.N., Sinelshchikov D.I., A note on "New kink – shaped solutions and periodic wave solutions for the (2+1) – dimensional sine – Gordon equation" Applied Mathematics and Computation, 216 (2010), 2479 – 2481
- [19] Parkes E.J., A note on travelling - wave solutions to Lax's seventh - order KdV equation, Appl. Math. Comput. 2009; 215:864 - 865
- [20] Parkes E.J., A note on solitary travelling-wave solutions to the transformed reduced Ostrovsky equation, Commun. Nonlinear. Sci. Numer. Simul., 15 (2010), 2769 – 2771